## Ptolemy's Sphere of Visible Stars

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In one of the volumes of his thirteen-part Mathematika Synthaxis, Claudius Ptolemy cataloged 1022 visible stars which could be seen from his home in Alexandria. That Alexandria lies at almost $30^{\circ} \mathrm{N}$ latitude means that Ptolemy would have been able to see about 7\%
 more stars had he been able to observe them from the equator. Correcting for that problem, the number would be closer to 1,100 . The argument can be made that the stars we can see without the aid of a telescope are the closest stars owing to their brightness and that they lie within a "sphere of visibility." It is instructive to estimate the size of Ptolemy's sphere of visible stars knowing what we know today about our place in the Milky Way Galaxy.

Today we know that our home is in one arm of the Milky Way, a spiral galaxy with an estimated diameter of 100,000 light years and containing 100 billion stars. It is a member of our "local group" which at this writing numbers a total of 45 galaxies. Four others are known as "nearby non-member(?)" galaxies. Our Milky Way is seen as a bright band across the sky on a clear night, but beyond a distance represented approximately by the boundary of Ptolemy's sphere of visible stars, individual stars are not discernible with the naked eye. Galileo's announcement in the Starry Messenger of 1610 that his nine-power telescope was able to discern thousands more stars than had been catalogued by Ptolemy, thus contradicting Aristotle's version of "what you see is what you get" as regards stars in the sky, caused a
 shock horror among the keepers of the traditional wisdom which can only be vaguely imagined today. We use the galaxy M51, above, to illustrate a spiral galaxy like our own. M51 is the famous
 Whirlpool galaxy, discovered by Charles Messier on October 13, 1773 when observing a comet. Its companion, NGC 5195, was discovered in 1781 by his friend, Pierre Mechain, so that it is mentioned in his 1784 catalogue. The barely visible wisp of trailing stars from each of the galaxies is referred to as "the atmospheres." The atmospheres seem barely to touch each other. M51 is the dominating member of a small group of galaxies. As it is about 37 million light years distant and so conspicuous, it is actually a big and luminous galaxy.


Here is M102 (left), a "local
group" galaxy which is convenient for our calculation of the diameter of Ptolemy's sphere of visible stars because we are viewing it on edge and its dimensions can be more easily estimated.

If we rotate it slightly, as at the right, then enlarge it and pick an enclosure which includes only the galaxy it is possible to estimate its thickness and diameter by assuming the enclosure to be a cylinder of diameter 100,000 light years and height 25,000 light years. (The diameter to height ratio of the enlarged figure is $4: 1$ ). Taking 1,100 stars to be a better figure for all visible stars in all directions from the earth, the volume represented by those stars would be given by the ratio
$\frac{V \text { cubic light years }}{\pi \times 50,000^{2} \times 25,000 \text { cubic light years }}=\frac{1100 \text { stars }}{100 \text { billion stars }}$

For this calculation, V turns out to be a little more than 2 million cubic light years. Using the equation below we can determine the radius of a sphere of volume 2 million cubic light years:

$$
V=\frac{4}{3} \pi r^{3}
$$

The value of $r$ for Ptolemy's sphere is calculated to be very nearly 78 light years giving us a sphere of diameter 156 light years. How big and where would Ptolemy's sphere be on the enlarged image above if that image were to represent the Milky Way galaxy? This calculation requires the use of the ratio of the diameter of Ptolemy's sphere to the diameter of the Milky Way.

$$
\frac{D \text { inches }}{4 \text { inches }}=\frac{156 \text { light years }}{100,000 \text { light years }}
$$

D turns out to be nearly 0.006 inches or about half the diameter of the period at the end of this sentence. Making the picture of the great galaxy of Andromeda just 4 inches as in the enlarged edge-on galaxy above, we can show the approximate position of Ptolemy's sphere. As an aside, we're using the great galaxy of Andromeda to illustrate where Ptolemy's sphere would be because Andromeda is about the same size as the Milky Way (1.5 times as large) and, like Andromeda, the Milky Way is believed also to be a spiral galaxy. Moreover, we don't have a photo of the whole Milky Way galaxy because we live inside it! The dot we have used to represent Ptolemy's sphere was already in the image and is about the right size to represent a sphere 156 light years in diameter immersed in a body which has a diameter approximately that of the Milky Way. Visible to the naked eye even under less than ideal conditions, Andromeda was known as the little cloud" to the Persian
 astronomer Al-Sufi, who described it in 964 AD in his Book of Fixed Stars; it must have been observed by Persian astronomers as early as 905 AD , or earlier. Charles Messier was obviously unaware of this early report and ascribed its discovery to Simon Marius, who was the first to give a telescopic description in 1612. Unaware of both Al Sufi's and Marius' discovery, Giovanni Batista Hodierna independently rediscovered this object sometime before 1654.

The foregoing represents only a very rough estimate based on some rather shaky assumptions, not the least of which is the size of the cylinder holding the Milky Way galaxy. In recent years it has become possible to measure distances to some of the brightest stars in the galaxy using the Doppler effect of frequency shifts due to relative velocities between our solar system and neighboring stars, then on the basis of angles of view and anticipated velocities of stars at certain distances from the center of the galaxy to estimate their distance from us. Here are data on the twenty-six brightest stars. The discrepancy between our calculation and these data invite our reflection on the matter of estimates with large uncertainties.

## The Twenty-Six Brightest Stars

| Common Name | Scientific Name | Distance in light years except for the sun |
| :--- | :--- | :--- |
|  |  |  |
| Sun |  | 8 light minutes, 0.0000152 light years |
| Sirius | Alpha CMa | 8.6 |
| Canopus | Alpha Car | 74 |
| Rigil Kentaurus | Alpha Cen | 4.3 |
| Arcturus | Alpha Boo | 34 |
| Vega | Alpha Lyr | 25 |
| Capella | Alpha Aur | 41 |
| Rigel | Beta Ori | $\sim 1400$ |
| Procyon | Alpha CMi | 11.4 |
| Achernar | Alpha Eri | 69 |
| Betelgeuse | Alpha Ori | $\sim 1400$ |
| Hadar | Beta Cen | 320 |
| Acrux | Alpha Cru | 510 |
| Altair | Alpha Aql | 16 |
| Aldebaran | Alpha Tau | 60 |
| Antares | Alpha Sco | $\sim 520$ |
| Spica | Alpha Vir | 220 |
| Pollux | Beta Gem | 40 |
| Fomalhaut | Alpha PsA | 22 |
| Becrux | Beta Cru | 460 |
| Deneb | Alpha Cyg | 1500 |
| Regulus | Alpha Leo | 69 |
| Adhara | Epsilon CMa | 570 |
| Castor | Alpha Gem | 49 |
| Gacrux | Gamma Cru | 120 |
| Shaula | Lambda Sco | 330 |
|  |  |  |

Excluding the sun, the average distance to these stars is 299 light years. Excluding the sun and those which have distances greater than 1000 light years (that is, the ones at both extreme, closest and furthest), the average becomes 144 light years. The rough estimate of the diameter of Ptolemy's sphere of visible stars disagrees by about a factor of 4 from the first average above and by a factor of 2 from the second average. Moreover, this list should remind us that the average distance assumes that some stars will be far beyond the that average.

Thanks to the many people whose hard work resulted in the World Wide Web pages referenced below, much material from which was extracted to prepare this short article.
http://www.astro.wisc.edu/~dolan/constellations/extra/brightest.html
http://www.seds.org/messier//m/m031.html
http://www.seds.org/messier//more/local.html
$\mathrm{http}: / / \mathrm{burro} . \mathrm{cwru} . e d u / \mathrm{SSAnims} /$ (This link offers some spectacular animated sequences of simulated galaxy collisions. The objective of one sequence or another is not always clear but in every case one is treated to the unpredictability of the change in shapes when one diffuse body the particles of which are subject to gravitational forces come into close proximity with another.
http://www.seds.org/messier//indexes.html (A fine visual index of the Messier Catalog)
http://www.seds.org/messier//xtra/history/biograph.html (A short biography of Charles Messier)
http://www.seds.org/messier/m/m051.html (A description of M51, the "Whirlpool Galaxy" and
its companion NGC5195)

